Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

DEPARTMENTS.

NOTE. All solutions of problems, problems for solution, and other department contributions should be sent direct to THE AMERICAN MATHEMATICAL MONTHLY, 1227 Clay Street, Springfield, Mo.

SOLUTIONS OF PROBLEMS.

The following problem was received too late for publication: Miscellaneous No. 150, solved by L. E. Newcomb.

ALGEBRA.

243. Proposed by **PROFESSOR WILLIAM HOOVER, Ph. D.**, Athens, Ohio.

Find the infinite root of $\frac{1}{x}+\frac{1}{a}=\sqrt{\left[\frac{1}{a^2}-\sqrt{\frac{1}{a^2x^2}+\frac{1}{x^4}}\right]}$.

Solution by **ELMER SCHUYLER, A. B.**, Brooklyn, New York.

Squaring the sides and reducing we have,

$$\frac{1}{x^2}+\frac{2}{ax}=-\sqrt{\frac{1}{a^2x^2}+\frac{1}{x^4}}.$$

Whence $1/x=0$, or $x=\infty$. Also,

$$\frac{1}{x}+\frac{2}{a}=-\sqrt{\frac{1}{a^2}+\frac{1}{x^2}}.$$

Squaring this and reducing, we get

$$\frac{1}{x}=-\frac{3}{4a}, \text{ or } x=-\frac{4a}{3}.$$

Since $\frac{1}{x^2}$ cancelled out on each side of the equation, $\frac{1}{x}=0$, or $x=\infty$. The only value of x satisfying the original equation, the radicals being taken with positive signs, is $x=\infty$.

Also solved by B. F. Finkel, G. W. Greenwood, J. E. Sanders, G. B. M. Zerr, and the Proposer.

AVERAGE AND PROBABILITY.

168. Proposed by **J. SCHEFFER, A. M.**, Hagerstown, Md.

Find the average area of a triangle two of whose sides have the constant sum $2a$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let x, y be the sides and θ the included angle.

Then $\frac{1}{2}xy\sin\theta=$area, and $x+y=2a$.

$$\therefore \text{Average area} = \triangle = \frac{\frac{1}{2}\int_0^{2a}\int_0^{\pi} x(2a-x)\sin\theta\, dx\, d\theta}{\int_0^{2a}\int_0^{\pi} dx\, d\theta}.$$

$$\therefore \triangle = \frac{1}{4\pi a}\int_0^{2a}\int_0^{\pi} x(2a-x)\sin\theta\, dx\, d\theta = \frac{1}{2\pi a}\int_0^{2a} x(2a-x)dx = \frac{2a^2}{3\pi}.$$

169. Proposed by HENRY HEATON, Atlantic, Iowa.

What is the average length of all straight lines that can be drawn within a given square?

I. Solution by B. F. FINKEL, A. M., Fellow in The University of Pennsylvania.

Let $ABCD$ be the given square, the length of whose side is a; and let EF and $E'F'$ be two parallel lines: EF terminating in two adjacent sides of the square, and $E'F'$ in two opposite sides. Let AH, $=x$, be the perpendicular to EF from A, and let the angle $FAH=\theta$.

Then $EF=\dfrac{x}{\sin\theta\cos\theta}$, and $E'F'=\dfrac{a}{\cos\theta}$.

Since the law of distribution of the lines is not given, we will assume that those having a given direction are uniformly distributed, and those passing through a given point are distributed so that the number of lines lying in a given angle is directly proportional to the magnitude of the angle.

Now, the limits of x are 0 and $AH'=a\sin\theta$ for $EF=x/\sin\theta\cos\theta$, and $AH'=a\sin\theta$ and $AH''=a\cos\theta$, for $EF=E'F'=a/\cos\theta$; and the limits of θ are 0 and $\frac{1}{4}\pi$. Then the average length of lines drawn within the square is

$$l_{\triangle} = \frac{\int_0^{\frac{1}{4}\pi} d\theta \left[2\int_0^{a\sin\theta} \sec\theta\cos\theta\, xdx + \int_{a\sin\theta}^{a\cos\theta} a\sec\theta\, dx\right]}{\int_0^{\frac{1}{4}\pi} d\theta \int_0^{\sqrt{2}\,a\cos(\frac{1}{4}\pi-\theta)} dx}$$

$$= \frac{1}{a}\int_0^{\frac{1}{4}\pi} d\theta \left[2\int_0^{a\sin\theta} \sec\theta\csc\theta\, xdx + a\int_{a\sin\theta}^{a\cos\theta} \sec\theta\, dx\right]$$

$$= \frac{1}{a}\int_0^{\frac{1}{4}\pi} d\theta \left[\sec\theta\csc\theta\left(x^2\right)_0^{a\sin\theta} + a\sec\theta\left(x\right)_{a\sin\theta}^{a\cos\theta}\right]$$

$$= \frac{1}{a}\int_0^{\frac{1}{4}\pi} a^2\, d\theta = \tfrac{1}{4}\pi a.$$